

# Towards a measure of auditory-filter phase response

Andrew J. Oxenham<sup>a)</sup>

Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

Torsten Dau<sup>b)</sup>

Hearing Research Center, Department of Biomedical Engineering, Boston University,  
44 Cummings Street, Boston, Massachusetts 02215

(Received 19 February 2001; revised 22 June 2001; accepted 4 September 2001)

This study investigates how the phase curvature of the auditory filters varies with center frequency (CF) and level. Harmonic tone complex maskers were used, with component phases adjusted using a variant of an equation proposed by Schroeder [IEEE Trans. Inf. Theory **16**, 85–89 (1970)]. In experiment 1, the phase curvature of the masker was varied systematically and sinusoidal signal thresholds were measured at frequencies from 125 to 8000 Hz. At all signal frequencies, threshold differences of 20 dB or more were observed between the most effective and least effective masker phase curvature. In experiment 2, the effect of overall masker level on masker phase effects was studied using signal frequencies of 250, 1000, and 4000 Hz. The results were used to estimate the phase curvature of the auditory filters. The estimated relative phase curvature decreases dramatically with decreasing CF below 1000 Hz. At frequencies above 1000 Hz, relative auditory-filter phase curvature increases only slowly with increasing CF, or may remain constant. The phase curvature of the auditory filters seems to be broadly independent of overall level. Most aspects of the data are in qualitative agreement with peripheral physiological findings from other mammals, which suggests that the phase responses observed here are of peripheral origin. However, in contrast to the data reported in a cat auditory-nerve study [Carney *et al.*, J. Acoust. Soc. Am. **105**, 2384–2391 (1999)], no reversal in the sign of the phase curvature was observed at very low frequencies. Overall, the results provide a framework for mapping out the phase curvature of the auditory filters and provide constraints on future models of peripheral filtering in the human auditory system. © 2001 Acoustical Society of America. [DOI: 10.1121/1.1414706]

PACS numbers: 43.66.Dc, 43.66.Nm [SPB]

## I. INTRODUCTION

One of the earliest and most fundamental stages of auditory processing is the frequency analysis that takes place in the cochlea. This stage has often been likened to a bank of overlapping bandpass filters, and much effort has gone into characterizing the magnitude response of these “auditory filters” (e.g., Fletcher, 1940; Zwicker *et al.*, 1957; Plomp, 1964; Patterson, 1976; Houtgast, 1977; Glasberg and Moore, 1990). In contrast, the phase response of the auditory filters has received much less attention. This may be due in part to the longstanding belief that the ear is essentially “phase deaf” (Helmholtz, 1954), and that in many situations auditory perception can be understood in terms of the power spectrum of a stimulus (i.e., by discarding the phase information). Although a number of studies have since shown that the ear is sensitive to changes in stimulus phase both within an auditory filter (Mathes and Miller, 1947; Zwicker, 1952; Goldstein, 1967) and, to a lesser extent, across filters (Patterson, 1987), they have mostly assumed either implicitly or explicitly that any effect of the auditory filters themselves on the stimulus phase can be ignored. In many circumstances, the phase response of the auditory system is indeed irrel-

evant. Also, there are only certain aspects of the phase response that have any psychophysically measurable influence. For instance, both the absolute phase,  $\theta$ , and the group delay,  $d\theta/df$ , are generally only meaningful in the context of a fixed time reference and cannot be estimated psychophysically (Goldstein, 1967; Kohlrausch and Sander, 1995). When defining individual auditory filters, the first term that has any psychophysical relevance is the phase curvature,  $d^2\theta/df^2$ , which is the rate of change of group delay as a function of frequency.

Following from the work of Smith *et al.* (1986), Kohlrausch and Sander (1995) provided instances in which it is clear that the phase response of the auditory filters cannot be ignored. They used equal-amplitude harmonic tone complexes with the phases set according to an equation proposed by Schroeder (1970), which have come to be known as Schroeder-phase stimuli. These stimuli are characterized by very flat temporal envelopes and can be thought of as repeating rising (Schroeder negative, or  $m_-$ ) or falling (Schroeder positive, or  $m_+$ ) linear frequency sweeps. It has been found that when these stimuli are used as maskers they can produce pure-tone masked thresholds that differ by more than 20 dB, with the  $m_+$  stimulus generally producing the lower threshold. Schroeder-phase stimuli have constant phase curvature, implying that the frequency sweep rate is constant (see Kohlrausch and Sander, 1995, Fig. 2). It has been proposed that the positive phase curvature of the  $m_+$  stimulus interacts

<sup>a)</sup>Electronic mail: oxenham@mit.edu

<sup>b)</sup>Present address: Medizinische Physik, Universität Oldenburg, 26111 Oldenburg, Germany. Electronic mail: torsten.dau@medi.physik.uni-oldenburg.de

with the phase curvature of the auditory filter to produce a highly modulated filtered waveform with a phase curvature close to zero, such as a sine-phase complex. This implies that the curvature of the auditory filter must be negative, so that it compensates in some degree for the positive curvature of the  $m_+$  masker. In the case of the  $m_-$  stimulus, the negative curvature of the stimulus is simply increased by the auditory filter, resulting in a filtered waveform that still has a relatively flat temporal envelope. The argument that signal thresholds are related to the degree of modulation of the masker waveform after auditory filtering was supported by the finding that  $m_+$  maskers generally produce much more modulated masking period patterns than do  $m_-$  maskers, suggesting that the  $m_+$  maskers produce highly modulated, or “peaky”, waveforms after cochlear filtering, while the response to  $m_-$  stimuli is characterized by much flatter filtered waveforms (Kohlrausch and Sander, 1995).

Psychophysical results from studies using Schroeder-phase stimuli in normal-hearing listeners have suggested that the phase curvature of the auditory filters is negative at all frequencies tested so far (Smith *et al.*, 1986; Kohlrausch and Sander, 1995; Carlyon and Datta, 1997a, b). This conclusion can be compared qualitatively with measures of cochlear phase response in the physiological literature. Evidence for negative phase curvature in the chinchilla peripheral auditory system has been found at the level of the basilar membrane (BM) (Ruggero *et al.*, 1997; Rhode and Recio, 2000). Similarly, in the guinea pig, negative curvature in the form of an upward frequency glide in the BM impulse response has been reported by de Boer and Nuttall (1997). While the sign of the curvature from these studies is consistent with that found psychophysically, a quantitative comparison is made difficult by the fact that all these BM data were recorded from the basal, or high-frequency, end of the cochlea, at characteristic frequencies (CFs) well above the signal frequencies studied in psychophysical experiments so far. Carney *et al.* (1999) analyzed the responses of auditory-nerve fibers in the cat, which allowed the examination of a wide range of CFs. They also found evidence of a frequency glide in the impulse response, indicating a nonzero phase curvature. However, an upward frequency glide, consistent with negative phase curvature, was found only for auditory-nerve fibers with CFs above about 1.5 kHz. Around 1 kHz, no glide was apparent, and below about 750 Hz a downward frequency glide was observed, indicating positive phase curvature at low CFs. A similar, albeit less clear, trend can be observed in the phase response curves from the cat auditory-nerve data of Pfeiffer and Molnar (1970). In contrast, auditory-nerve data from the squirrel monkey seem to indicate no reversal of phase curvature at low CFs; instead, data from CFs of 200 and 1000 Hz both show essentially zero phase curvature (Anderson *et al.*, 1970). In some of the few BM data available from the apical turn of the cochlea (Cooper and Rhode, 1996), there appears to be no significant curvature in the phase response of the guinea pig at a place along the BM with a CF of around 400 Hz. Due to the sparsity of data, however, it is not clear whether these differences represent true species differences or simply measurement uncertainty.

The data from Carney *et al.*, together with other phase data from the BM or auditory nerve, have recently been analyzed by Shera (2001a). He concluded that, once normalized for CF, the frequency glides in the impulse responses were relatively constant across different species and across CF, at least above about 1.5 kHz. He referred to the regions above and below 1.5 kHz (in the cat) as the “scaling” and “non-scaling” regions, respectively. The cat’s hearing extends to approximately 50–60 kHz. If we assume that the mechanics of the cat and human cochlea are similar to within a scale factor (Greenwood, 1990), the transition frequency of between 1 and 1.5 kHz found by Carney *et al.* in the cat may correspond to a frequency of around 300 to 500 Hz in humans. No behavioral estimates of phase curvature have been made below 500 Hz in humans so far. Furthermore, behavioral estimates of phase curvature at higher frequencies have not been sufficiently extensive for any general conclusions to be drawn on whether the phase response in humans scales with CF at higher frequencies.

The present study had two main aims. The first aim was to estimate the phase curvature of the auditory filters over a wide range of center frequencies. If the mechanics of the human cochlea are similar to those of the cat, a reversal in the sign of the phase curvature might be expected at very low signal frequencies. More generally, a knowledge of the phase properties of the auditory filters will be important in modeling the human auditory periphery. As pointed out by others (e.g., de Boer and Nuttall, 1997), the temporal properties of the BM responses in various animals provide very strong constraints for models of cochlear mechanics. The second aim of the study was to investigate whether, and how, phase curvature changes as a function of level. One consistent feature of physiological measures of phase curvature is their level invariance (for a recent review and theoretical implications, see Shera, 2001b). This feature has not yet been tested behaviorally in humans.

## II. EXPERIMENT 1. EFFECTS OF SIGNAL FREQUENCY ON ESTIMATED PHASE CURVATURE

### A. Methods

#### 1. Stimuli

Thresholds were measured for a sinusoidal signal in the presence of a harmonic tone complex masker. The masker had a total duration of 320 ms and was gated on and off with 30-ms raised-cosine ramps. The signal was temporally centered within the masker and had a total duration of 260 ms, gated with 50-ms raised-cosine ramps. The signal frequency ( $f_s$ ) was selected from octave frequencies between 125 and 4000 Hz. The signal was added to the masker with a starting phase that was selected randomly from trial to trial. The masker always comprised components between  $0.4f_s$  and  $1.6f_s$ . This bandwidth was chosen because Oxenham and Dau (2001) found that only components within this range contributed to the threshold of a 2-kHz signal in an  $m_+$  masker. The phases of the components were selected according to a modification of Schroeder’s (1970) equation, as suggested by Lentz and Leek (2001):

$$\theta_n = C\pi n(n-1)/N, \quad -1 \leq C \leq 1. \quad (1)$$

A Schroeder positive ( $m_+$ ) or Schroeder negative ( $m_-$ ) complex is generated when  $C=1$  or  $C=-1$ , respectively. When  $C=0$ , a sine-phase complex is generated. The phase curvature of the complex was

$$\frac{d^2\theta}{df^2} = C \frac{2\pi}{Nf_0^2}. \quad (2)$$

By varying the  $C$  value from  $-1$  to  $1$ , a range of masker phase curvatures, or frequency sweep rates, can be generated (see also Schreiner *et al.*, 1983, and Mehrgardt and Schroeder, 1983). In one set of conditions, signal frequencies of 125, 250, 500, and 1000 Hz were tested and the masker fundamental frequency ( $f_0$ ) was set to  $0.1f_s$ . At signal frequencies of 250 and 1000 Hz, two other masker  $f_0$ 's were also tested: 12.5 and 50 Hz at 250 Hz, and 25 and 50 Hz at 1000 Hz. At signal frequencies of 2000 and 4000 Hz, the masker  $f_0$  was set to 50 Hz. Signal thresholds were measured for values of  $C$  between  $-1$  and  $1$  in steps of  $0.25$ . The overall level of the masker was 75 dB SPL. For the most common case of 13 components (when  $f_0=0.1f_s$ ) this corresponded to a level of about 64 dB SPL per component.

The stimuli were generated digitally and played out using a LynxOne (LynxStudio) sound card with 16-bit resolution at a sampling rate of 32 kHz. The stimuli were passed through a programmable attenuator (TDT PA4) and headphone buffer (TDT HB6) before being presented to the listener's left ear via Sennheiser HD 580 headphones. Listeners were seated in a double-walled sound-attenuating booth.

## 2. Procedure

An adaptive three-interval, three-alternative, forced-choice procedure was used in conjunction with a 2-down, 1-up tracking rule to estimate the 70.7%-correct point on the psychometric function (Levitt, 1971). Each interval in a trial was separated by an interstimulus interval (ISI) of 500 ms. The intervals were marked on a computer monitor and feedback was provided after each trial. Listeners responded via the computer keyboard or mouse. The initial step size was 5 dB, which was reduced to 2 dB after the first four reversals. Threshold was defined as the mean of the remaining six reversals. Three threshold estimates were initially made for each condition. If the standard deviation across the three runs was greater than 4 dB, another three estimates were made and the mean of all six was recorded. The conditions were run using a randomized blocked design, with all conditions within one repetition being presented before embarking on the next repetition. The order of presentation of the conditions was selected randomly for each listener and each repetition. Once a signal frequency and masker  $f_0$  had been selected, all the  $C$  values for that condition were tested in random order before the next condition was run. Measurements were made in 2-h sessions, including many short breaks. No more than one session per listener was completed in any one day.

## 3. Listeners

The four listeners (one male—PG) were all paid for their participation. Their ages ranged from 20 to 29 years (mean

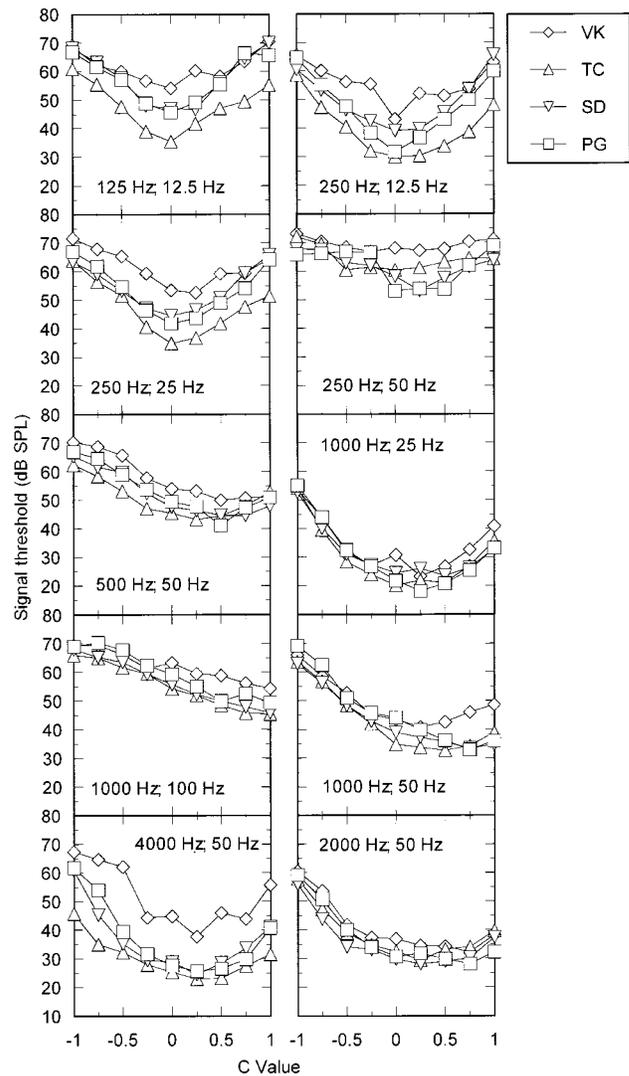


FIG. 1. Individual data from all the conditions tested in experiment 1. The first and second number in each panel denote the signal frequency and the fundamental frequency of the masker, respectively.

23 years). All had absolute thresholds of less than 15 dB HL at octave frequencies between 250 and 8000 Hz and had previously taken part in a similar experiment. All were given between 1 and 2 h further practice before data were collected.

## B. Results

The individual data from all conditions are shown in Fig. 1, with signal threshold plotted as a function of masker  $C$  value. The first and second numbers in the panel captions denote the signal frequency and the masker  $f_0$ , respectively. There are some considerable individual differences in masked threshold values. However, the trends in the data are very similar across listeners. For this reason, the discussion of the data will refer solely to the mean values.

In interpreting these data, it is assumed that the minimum in masking is achieved when the phase curvature of the stimulus is equal in magnitude but opposite in sign to the phase curvature of the auditory filter centered at the signal frequency. In this way, the minimum threshold is achieved

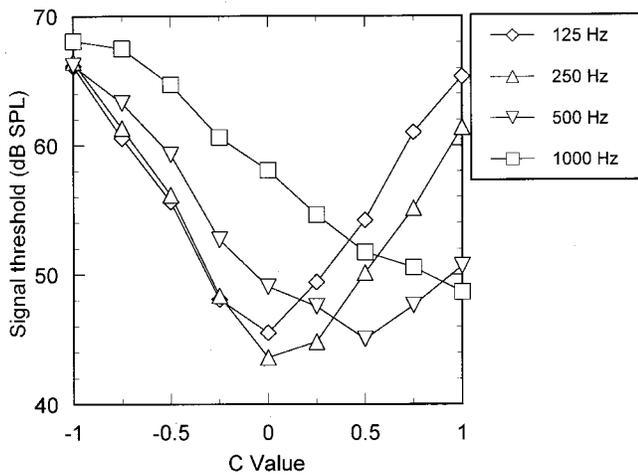


FIG. 2. Mean data showing masked thresholds as a function of masker phase curvature at four signal frequencies. The masker bandwidth and fundamental frequency were always proportional to the signal frequency.

when the filtered waveform most closely resembles a sine-phase complex and is maximally peaky. In support of this assumption, Kohlrausch and Sander (1995) have shown a strong correlation between the peakiness of a filtered waveform (as estimated using masking period patterns) and the threshold for a long-duration sinusoidal signal in a given masker. The second assumption is that the phase curvature of the auditory filters can be approximated as constant within the filter passband. This assumption seems to be valid for all mammalian species studied so far. For instance, Carney *et al.* (1999) found that the instantaneous frequency glides from their cat auditory-nerve impulse responses were well fitted with straight lines, thereby implying a constant phase curvature. This conclusion is also supported by de Boer and Nuttall's (1997, 2000a, b) guinea pig BM data, as analyzed by Shera (2001a). A plot of group delay as a function of frequency at any given CF (Shera's Fig. 4) shows that the group delay can be reasonably well approximated by a straight line for frequencies above about half an octave below CF. A straight line on these coordinates implies constant curvature. Finally, it is important to note that, in contrast to stimuli with maximally "flat" envelopes (Schroeder, 1970; Hartmann and Pumplin, 1988), the phase relationships producing the maximally peaky envelope do not depend on the magnitudes of the components. In other words, for a given masker, the magnitude spectrum of the auditory filter, the headphones, and the outer/middle-ear transform, should not affect the value of  $C$  at which the minimum threshold occurs.

The data are discussed in the following three sections. The first examines conditions in which the  $f_0$  and masker bandwidth were proportional to the signal frequency; the second examines the effects of changing  $f_s$  while  $f_0$  is held constant at 50 Hz; the third examines the effects of changing  $f_0$  while keeping  $f_s$  fixed at 250 or 1000 Hz.

### 1. Effects of signal frequency with a variable $f_0$ : A test of scaling symmetry at low CFs

Figure 2 shows the mean data for signal frequencies of 125, 250, 500, and 1000 Hz. The masker  $f_0$  was always

$0.1f_s$ . Thresholds are plotted as a function of  $C$  value, and the different symbols represent different signal frequencies, as shown in the legend. There are large variations in threshold as a function of masker phase at all signal frequencies; all curves have maximum masking differences of at least 20 dB. As the total number of masker components is constant in these four conditions ( $N=13$ ), the phase curvature of the masker is inversely proportional to the square of the signal frequency [see Eq. (2)]. This inverse-square relationship is also what would be expected of filters with scaling symmetry (Kohlrausch and Sander, 1995; Shera, 2001a). In other words, if the auditory filters were of constant shape and bandwidth when plotted on a logarithmic frequency axis (e.g., constant- $Q$  filters), their phase curvature would be expected to vary inversely with the square of the filter CF. Therefore, if scaling symmetry applied to auditory filters with CFs between 125 and 1000 Hz, all four curves shown in Fig. 2 should have the same value of  $C$  at the point of minimum masking. This is clearly not the case. Instead, the masking minimum at 1 kHz occurs at (or above)  $C=1$ , and there is a rapid trend for the  $C$  value to decrease with decreasing signal frequency, such that at 125 and 250 Hz, the minimum appears to lie at or close to  $C=0$ . Interestingly, at 125 and 250 Hz, the functions are very similar for  $C<0$ , but diverge somewhat for  $C>0$ . This point is discussed further in Sec. IV.

In some respects the results are consistent with physiological estimates of cochlear phase response. Shera has pointed out that approximate scaling symmetry appears to hold at frequencies above about 1.5 kHz in the cat, but not below. As the signal frequencies in Fig. 2 were all below 1.5 kHz, one might not expect to find scaling symmetry in the phase curvature. In other respects, however, our data are not consistent with physiological findings, at least in the cat. As mentioned in the Introduction, Carney *et al.* (1999) found that the sign of the phase curvature reversed at frequencies below about 1000 Hz. Such a reversal would be manifest by negative  $C$  values at the point of minimum masking. Our data, down to 125 Hz, show no evidence for such a reversal. Instead, the phase curvature tends to zero at the lowest signal frequencies. It is not clear whether this difference is due to the different techniques (physiological vs behavioral) or whether it reflects true species differences. As mentioned in the Introduction, the fact that the available data from squirrel monkey (Anderson *et al.*, 1970) also seem to show zero curvature at low CFs leaves open the possibility of species differences.

### 2. Effects of signal frequency with a fixed masker $f_0$ : A test of scaling symmetry at higher CFs

Figure 3 shows the mean data for the five signal frequencies, including 2000 and 4000 Hz, tested with a fixed masker  $f_0$  of 50 Hz. A general trend is that the maximum masking difference increases with increasing signal frequency. This is expected, as the bandwidth of the auditory filters increases with increasing CF while, in this case, the  $f_0$  or component density remains constant. For these conditions, a doubling of  $f_s$  leads to a doubling of  $N$ , as the absolute

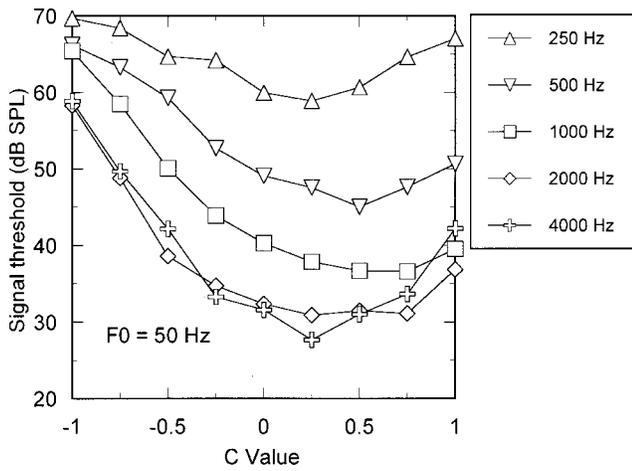


FIG. 3. Mean data showing masked thresholds as a function of masker phase curvature at five signal frequencies, from 250 to 4000 Hz. The masker bandwidth was proportional to the signal frequency, but the masker fundamental frequency was held constant at 50 Hz.

bandwidth of the masker doubles. Therefore, if the auditory filters exhibited scaling symmetry, then the  $C$  value at the point of minimum masking should double with every halving of  $f_s$ . There is some indication of this type of relationship between 1000 and 4000 Hz, in that the  $C$  value of the minimum decreases with increasing  $f_s$ . However, as in Fig. 2, this relationship breaks down for  $f_s$  lower than 1000 Hz, with the  $C$  value at the minimum actually moving in the opposite direction. The results from the two highest signal frequencies provide some indication that auditory filters with CFs above about 1000 Hz may exhibit some form of scaling symmetry, as has also been found in the cat above about 1.5 kHz. However, as this conclusion is based on only two signal frequencies it must be treated with some caution. A further condition at a signal frequency of 8 kHz was tested on a new group of listeners after the original data had been collected. These data are described in Sec. II C.

### 3. Effects of masker $f_0$ : A test of consistency

If the point of minimum masking is determined by the interaction between the phase curvature of the masker and the auditory filter then, for a given signal frequency, the masker  $C$  value at the threshold minimum should vary with masker  $f_0$  in a predictable manner. Specifically, a doubling of  $f_0$  should lead to a doubling of the  $C$  value at the minimum. (As the masker bandwidth is fixed,  $N$  is approximately inversely proportional to  $f_0$ , leading to the phase curvature being inversely proportional to  $f_0$ , rather than  $f_0^2$ .) In this way the data provide a consistency test of the hypothesis underlying this work, as well as providing multiple estimates of phase curvature at a given signal frequency.

Figure 4 shows the mean data for signal frequencies of 250 Hz (left panel) and 1000 Hz (right panel) as a function of masker  $C$  value. The different symbols denote different masker  $f_0$ 's. Data in the right panel ( $f_s = 1000$  Hz) are in broad agreement with expectations: a doubling of  $f_0$  from 25 to 50 Hz leads to an increase in the  $C$  value at the minimum from about 0.25 to between 0.5 and 0.75. Another doubling from 50 to 100 Hz leads to the minimum point being found

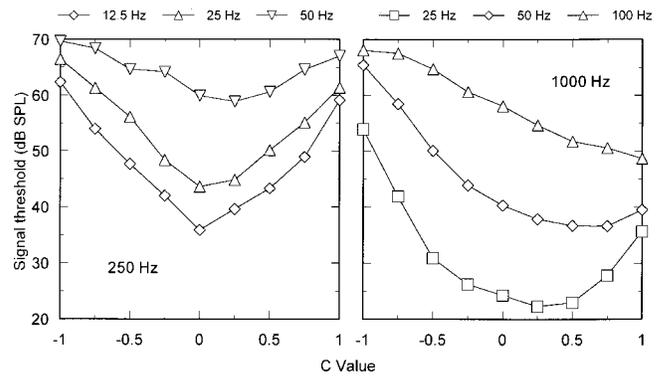


FIG. 4. Mean thresholds as a function of masker curvature at signal frequencies of 250 Hz (left panel) and 1000 Hz (right panel). Values given in the legend represent the masker fundamental frequencies tested.

at  $C = 1$  (or above). Data in the left panel (250 Hz) show  $C$  values of 0 at the minimum for  $f_0$ 's of 12.5 and 25 Hz, but a minimum of around 0.25 for an  $f_0$  of 50 Hz. This is also consistent with the above prediction, and suggests that the “true” minima at the two lower  $f_0$ 's lie somewhere between  $C = 0$  and  $C = 0.25$ . Finally, the data in Fig. 4 replicate the earlier finding (Kohlrausch and Sander, 1995) that increases in masker  $f_0$  lead to decreases in the maximum masking differences observed.

Supplemental experiments were carried out with signal frequencies of 125 and 250 Hz, using a number of masker  $C$  values between  $-0.25$  and  $0.25$ , in an effort to more closely define the point of minimum masker. However, the resulting masking functions were too flat to specify the minimum with any degree of accuracy.

### C. Masking with a signal frequency of 8 kHz

The data in Fig. 3 indicate that the phase curvature of the auditory filters may scale with CF for signal frequencies above about 1 kHz. However, this conclusion is based only on data from 2 and 4 kHz. To extend these findings, data were collected from four new listeners, aged between 23 and 32 years, who all had normal hearing as defined in experiment 1 and were given at least 2 h training. Each threshold reported is the mean of four estimates. The maskers had the same relative bandwidth as in the original experiment, extending from  $0.4$  to  $1.6f_s$ . The new listeners were tested on a condition with  $f_s = 8000$  Hz and  $f_0 = 200$  Hz. This is similar to the earlier 2-kHz condition, but scaled upwards by a factor of 4. If scaling symmetry holds for the phase response of the auditory filters between 2000 and 8000 Hz, then the  $C$  value producing the minimum threshold should be similar for these two conditions.

The individual (symbols) and mean (solid curve) results from this experiment at 8 kHz are shown in Fig. 5, together with the mean results at 2 kHz, replotted from Fig. 3 (dashed curve). As in previous experiments, there are some considerable individual differences. The individual masking minima tend to lie between  $C$  values of 0.25 and 0.75, which is similar to the range found at 2 kHz. Thus, the data do not rule out the possibility that the auditory filter's phase curvature scales with CF above about 1 kHz, although the vari-

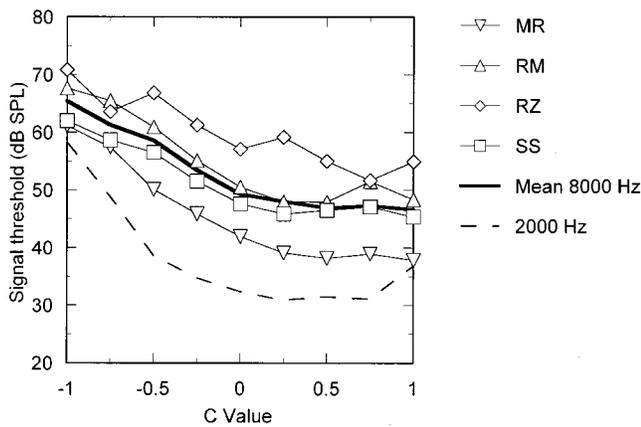


FIG. 5. Individual (symbols) and mean (solid curve) data for a signal frequency of 8000 Hz. The dashed curve denotes mean data with a 2-kHz signal, replotted from Fig. 3.

ability in the data tempers any strong conclusions. Nevertheless, there are certainly no gross deviations from the assumption of scaling symmetry, such as those observed at the lower signal frequencies.

#### D. Discussion: Role of peripheral compression

Experiment 1 shows that the interaction between the auditory-filter phase curvature and that of the stimulus plays a large role across a very wide range of signal frequencies. The finding of large masking differences even at the lowest frequencies is noteworthy. Physiological and psychophysical data have suggested that compression in the input-output function of the basilar membrane may be reduced or even absent at low CFs (Cooper and Yates, 1994; Hicks and Bacon, 1999; Plack and Oxenham, 2000). It has been shown using the temporal-window model that when some temporal smoothing occurs, peripheral compression is essential in producing large  $m_-/m_+$  masking differences (Oxenham and Dau, 2001). Because of this, the fact that hearing-impaired listeners seem to show little or no phase effects in masking (Summers and Leek, 1998; Summers, 2000) has been interpreted as evidence for the importance of peripheral compression. The present data therefore lead to the conclusion either that compression is still present at low frequencies, or that compression is not necessary to account for the data at low frequencies.

A review of the literature on compression at low frequencies suggests that it may be premature to rule out the possibility that peripheral compression is present at very low frequencies. There are very few reliable data of BM motion in the apical, or low-frequency, end of the cochlea, due to the technical difficulties of measuring there without causing structural damage (Cooper and Rhode, 1996). All the other measures, physiological (Cooper and Yates, 1994) and psychological (Hicks and Bacon, 1999; Plack and Oxenham, 2000), rely on indirect measures. These measures assume that the compression is frequency selective, and that frequencies well below CF are processed linearly. As pointed out by Plack and Oxenham (2000), if the compression at low frequencies were not frequency selective, i.e., if all frequencies were compressed equally, then none of the measures used so

far would reveal this compression. There are some physiological indications that this may be the case. For instance, Rhode and Cooper (1996) reported some nonlinear gain in the apical turn of the chinchilla cochlea but showed that, unlike in the basal turn, the gain did not appear to be frequency selective.

On the other hand, the  $f_0$ 's used in conjunction with the lowest signal frequencies are so low that the period of the stimulus may be large compared with the duration of the temporal window, or smoothing function used to characterize auditory temporal resolution. For instance, at 125 Hz, the  $f_0$  of 12.5 Hz corresponds to a period of 80 ms. This is considerably greater than the hypothesized duration of the temporal window, which is generally assumed to be around 10 ms (e.g., Oxenham and Moore, 1994). As shown by Oxenham and Dau (2001), when the temporal window is small compared with the period of the masker, large masking differences can be predicted without compression, and compression ceases to make much difference in the predictions.

In summary, the large differences in threshold at the lowest signal frequencies do not necessarily imply the presence of peripheral compression. However, they do suggest that hearing-impaired listeners should exhibit large masking differences due to phase changes if sufficiently low  $f_0$ 's are employed. Preliminary data indicate that hearing-impaired listeners in fact continue to show reduced effects of masker phase even at very low  $f_0$ 's (Dau and Oxenham, 2001). If confirmed, this finding may provide indirect evidence for peripheral compression at low CFs in normal hearing.

### III. EXPERIMENT 2. EFFECTS OF MASKER LEVEL ON ESTIMATED PHASE CURVATURE

The results from experiment 1 showed large effects of masker phase at all signal frequencies tested. Note, however, that the results would have looked rather different if only  $m_+$  and  $m_-$  waveforms had been used, corresponding to  $C=1$  and  $C=-1$ , respectively. For instance, no phase effects at a signal frequency of 125 Hz would have been observed at all (see Fig. 2). Earlier studies have shown that masking differences between  $m_-$  and  $m_+$  maskers decrease at low levels (Carlyon and Datta, 1997a, b) and are also somewhat reduced at very high levels (Summers and Leek, 1998). It has been hypothesized that this may be due to a reduction in peripheral compression, a narrowing of the auditory-filter bandwidth (at low levels), or a combination of these (Carlyon and Datta, 1997a). While quantitative simulations support both of these hypotheses (Oxenham and Dau, 2001), it remains possible that the phase response of the auditory filters changes with level in such a way that the differences between the filtered  $m_-$  and  $m_+$  waveforms are reduced at low and high levels. In other words, the lack of an  $m_-/m_+$  difference at low levels may be due to the two particular phase relationships chosen by past experimenters, rather than to a lack of any phase effects at low levels *per se*.

A second rationale for examining the effects of level stems from physiological results, which suggest that phase curvature in the mammalian cochlea is level invariant (de Boer and Nuttall, 1997; Recio *et al.*, 1998; Carney *et al.*, 1999; de Boer and Nuttall, 2000b). This finding is somewhat

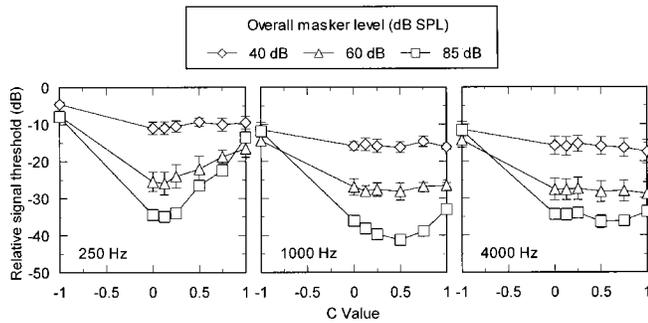


FIG. 6. Mean masked thresholds, relative to the masker component level, as a function of masker curvature at three signal frequencies (250, 1000, and 4000 Hz). Error bars denote  $\pm 1$  se of the mean across the four listeners. The different symbols denote different overall masker levels.

counterintuitive. In most physical filters, bandwidth and phase response covary. The fact that this seems not to be the case for the mammalian cochlea provides strong constraints for models of peripheral auditory processing (Shera, 2001b). The present experiment was designed to examine whether the same level invariance could be found for the phase curvature in human auditory filters.

### A. Methods

The stimuli and procedures were the same as in experiment 1 unless otherwise stated. The masker bandwidth again extended from  $0.4$  to  $1.6f_s$ . The masker  $f_0$  was 25, 50, and 100 for signal frequencies of 250, 1000, and 4000, respectively. Each masker was presented at overall levels of 40, 60, and 85 dB SPL. At 250 Hz, this corresponded to a masker component level 11.1 dB lower than the overall level; at 4000 Hz, this difference was 16.7 dB. Equation (1) was used to generate the phases of the masker components, with  $C$  values of  $-1$ ,  $0$ ,  $0.125$ ,  $0.25$ ,  $0.5$ ,  $0.75$ , and  $1$  tested. In pilot tests, it was noticed that distortion products became audible for the highest masker level at 4000 Hz. For this reason, a low-pass Gaussian white noise with a cutoff frequency of 1200 Hz was added to all the 4000-Hz maskers at a spectrum level 15 dB below the average spectrum level of the masker. This was sufficient to mask any audible distortion products. Four female listeners aged between 20 and 43 were paid for their participation in this experiment. One (TC) had taken part in experiment 1. The other three were given at least 2 h practice before data were collected. Their hearing was normal as defined in experiment 1. Each threshold reported is the mean of at least three threshold estimates. If after three estimates the standard deviation was greater than 4 dB, a further three measurements were made and the mean of all six estimates was recorded.

### B. Results and discussion

The mean results are shown in Fig. 6. The error bars represent  $\pm 1$  standard error of the mean. As the signal levels used in this experiment cover a very wide range in terms of dB SPL, Fig. 6 plots signal thresholds relative to masker component level for ease of comparison. The results were similar across listeners, both in terms of general trends and absolute values. It can be seen that relative thresholds are

generally lower and vary more with phase at the highest masker level than in the other two conditions. The signal-to-masker ratio at threshold is roughly constant with overall masker level only for the conditions producing the most masking (generally  $C = -1$ ). For other  $C$  values, the threshold signal-to-masker ratio varies with masker level by as much as 25 dB. These very large changes in signal-to-masker ratio imply a very gradual growth of masking, more in line with that normally found in forward masking than in simultaneous masking. This is consistent with the idea that thresholds in highly modulated maskers are governed by forward masking of the portions of the signal in the masker's temporal valleys, produced by the masker's temporal peaks (Bacon and Lee, 1997; Oxenham and Plack, 1998).

At the lowest masker level, the fact that thresholds vary so little with masker phase makes it difficult to draw strong conclusions about whether the phase curvature varies with level. However, it is clear that the failure of others (e.g., Carlyon and Datta, 1997b) to find large phase effects at low levels was not due to simply the wrong choice of phases. Instead, it appears that the reduction in phase effects at low levels may indeed be a consequence of reduced peripheral compression and/or narrow filter bandwidth. Where masking minima are apparent in the lower masker-level conditions, they appear to occur at the same  $C$  values as at the highest masker level. This suggests that phase curvature does not vary greatly with stimulus level. This is consistent with the findings from impulse responses of auditory-nerve fibers (Carney *et al.*, 1999) and from direct measurements of BM response (de Boer and Nuttall, 1997; Recio *et al.*, 1998).

One caveat should be borne in mind when comparing these data to those from physiological studies. As with studies of auditory-filter shape, we assume for simplicity that the use of a single frequency corresponds to estimating the response of a single CF region. However, due to the basal shift of the peak of the BM traveling wave at high levels, it is likely that the place maximally stimulated by a given frequency shifts somewhat with level. Consistent with all previous auditory-filter studies, however, we assume that the effects of this are of second-order importance. If the accuracy of our phase measurements warranted it, it would be a relatively trivial matter to take the assumed shifts of the traveling wave into account by altering the signal frequency accordingly.

## IV. ESTIMATING THE PHASE CURVATURE OF THE AUDITORY FILTERS

Under the assumptions discussed in Sec. II B, the data from these experiments can be used to estimate the phase curvature of the auditory system at octave frequencies between 125 and 8000 Hz. Specifically, it is assumed that the phase curvature in the passband of the auditory filter is equal and opposite to that of the masker at the point of minimum masking. The same technique was used by Lentz and Leek (2001). However, they used only one masker bandwidth (200–5000 Hz) with one  $f_0$  (100 Hz) and tested signal frequencies of 1, 2, 3, and 4 kHz.

The minimum  $C$  values (and hence the filter curvatures) were estimated across listeners and conditions from experi-

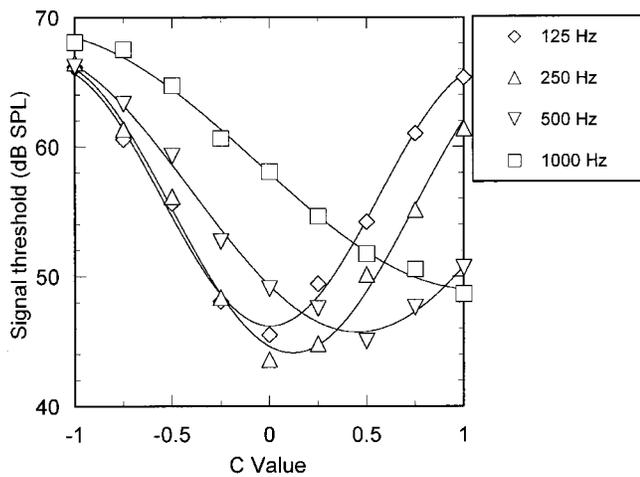


FIG. 7. Data replotted from Fig. 2 with curve fits using a sinusoidal function with four free parameters per condition [ $y = A \sin(\omega x + p) + C$ ]. The minima of the fitted functions were used to estimate the auditory-filter curvature in method 2 (see the text for details).

ment 1 using two methods. Method 1 involved finding the  $C$  value that produced the lowest masked threshold for each listener individually in each condition. This value was then converted into a value of curvature using Eq. (2). The mean of these values for each signal frequency was then used to estimate the curvature of the auditory filter centered at that frequency. The number of estimates varied with signal frequency. Specifically, at 125, 500, 2000, 4000, and 8000 Hz, only one condition was tested with four listeners, meaning that only four estimates of the curvature were available at each CF. In contrast, at 250 and 1000 Hz, more estimates were available, as three  $f_0$ 's were tested, giving 12 individual estimates. In the case of the 1000-Hz signal with the 100-Hz  $f_0$ ,  $C = 1$  was chosen as the minimum for all listeners, even though it is possible that the "true" minimum may have occurred at a slightly higher  $C$  value. Eliminating this point did not markedly change the predictions. Overall, the advantage of this method is that it makes no assumptions regarding the form of the data. The disadvantage is that the estimates rely only on points around the minimum of the function and make no use of other data points.

Method 2 involved fitting a function to the mean data in every condition and calculating the  $C$  value at the minimum of the fitted function. A number of mathematical functions were tried. A sinusoidal function was found to provide a very good fit to all conditions. The percentage of variance accounted for by this function was always greater than 95% and in most cases was greater than 99%. Examples of the fits are shown in Fig. 7. The symbols are data replotted from Fig. 2; the curves are the best-fitting (in a least-squares sense) sinusoidal functions. The functions capture the main trends of the data. Other less obvious trends are also well described including, for instance, the divergence of thresholds between the 125- and 250-Hz conditions for  $C$  values greater than 0. In cases with more than one condition for a given signal frequency, the  $C$  values of the function minima were averaged. The main advantage of using a mathematical function is that all the data contribute to estimating the minimum point of the function. The disadvantage is that a function

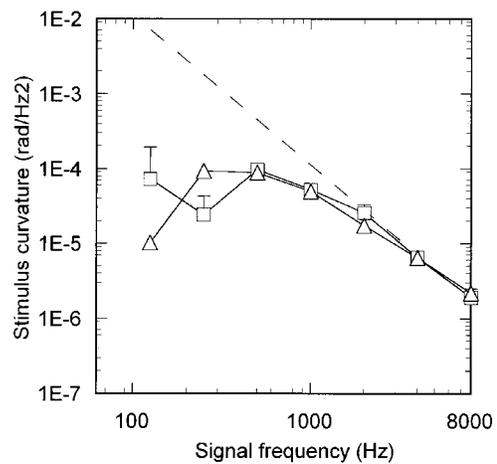


FIG. 8. Mean masker phase curvature producing masking minima, as a function of frequency. Squares and triangles represent estimates using methods 1 and 2, respectively (see the text for details). The slope of the dashed line shows the expected relationship if the auditory filters exhibited scaling symmetry. The error bars for method 1 represent  $+1$  se of the mean and are shown where they exceed the size of the symbol. For method 2, error bars could be calculated only for signal frequencies of 250 and 1000 Hz, and in both cases did not exceed the size of the symbol.

makes certain assumptions about the form of the data (e.g., that it is symmetric about the minimum point), which may not be justified. Because both methods have their strengths and weaknesses, the results from both are presented here.

Figure 8 shows the masker phase curvatures producing a masking minimum, estimated using method 1 (squares) and method 2 (triangles). The estimated phase curvature of the auditory filters has the same magnitude as the data shown in the figure, but with the opposite sign. The slope of the dashed line represents the pattern of results that would be expected if scaling symmetry prevailed throughout the frequency range tested. On first inspection, it seems that scaling symmetry may hold for frequencies above 1000 Hz, but clearly not below. In order to pursue the issue of scaling symmetry further, the estimated phase curvatures were transformed into dimensionless units by multiplying the curvature (originally in units of  $\text{rad}/\text{Hz}^2$ ) by  $f_s^2/2\pi$  (Shera, 2001a). When expressed in this way, for filters with scaling symmetry the resulting quantity should be constant and independent of CF. The transformed data from Fig. 8 are shown in Fig. 9. The other symbols denote estimates from earlier studies, discussed below. For frequencies above 250 Hz, the two methods are in reasonably good agreement. At the two lowest frequencies, however, the estimates differ considerably. It is interesting to note that the estimated standard errors for method 1 are also greatest at these two frequencies. At 125 Hz, zero phase curvature is less than 1 standard error below the mean from method 1, which is why it is not possible to plot the lower error bars on these logarithmic axes. The pattern of results is in broad agreement with our earlier conclusion that the phase curvature of the auditory filters may exhibit scaling symmetry at CFs from 2 to 8 kHz but departs severely from that pattern below about 1 kHz. However, at least using method 2, there is a trend towards an orderly

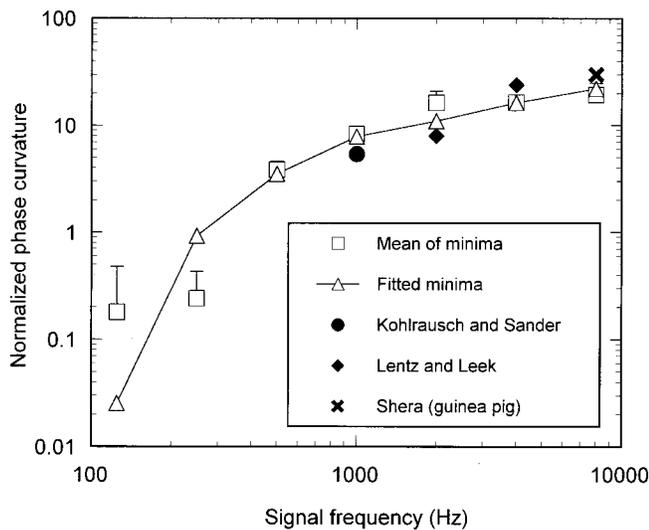


FIG. 9. Data from Fig. 8, transformed into dimensionless units by multiplying by  $f_s^2/2\pi$ . The filled symbols represent estimates from other studies, as indicated in the legend.

increase in relative curvature with increasing CF, as shown by the connecting lines, even at the higher signal frequencies.

It is possible to compare our estimates of phase curvature with those from two other studies in the literature. Converted into dimensionless units, Kohltrausch and Sander's (1995) estimate of auditory-filter phase curvature at 1100 Hz lies between  $-4.5$  and  $-6.3$  (the mean value is plotted as a filled circle in Fig. 9). Our estimates of  $-8.3$  (method 1) and  $-7.8$  (method 2) at 1000 Hz lie outside that range, but are within a factor of 2, corresponding to a change in  $C$  value from, for instance, 0.25 to 0.5. The range of masker phase curvatures used by Lentz and Leek (2001) did not extend sufficiently high to provide an accurate estimate of phase curvature at 1000 Hz. At 2000 and 4000 Hz, their estimated curvatures are around  $-8$  and  $-24$  (see the diamonds in Fig. 9), which are somewhat below and above our estimates, respectively. It is not clear what accounts for these discrepancies, although they may in part be due to individual differences. Also, as the measurement accuracy provided by this method is somewhat limited, a discrepancy by a factor of 2 can probably still be described as within the bounds of measurement uncertainty.

A recent analysis of BM and auditory-nerve phase-response data (Shera, 2001a) allows a cursory quantitative comparison of our results with BM data from the guinea pig. Shera's Fig. 4(b) plots group delay as a function of frequency in dimensionless units for a 17-kHz CF. The slope of the function can be reasonably approximated as constant for frequencies above about 0.7 CF. A linear regression of the function between 0.75 and 1.1 CF provides an estimate of 30 for the phase curvature in dimensionless units. In terms of place along the BM, a 17-kHz CF in the guinea pig corresponds roughly to an 8-kHz CF in human (Tsuji and Liberman, 1997). Thus, the physiological estimate from guinea pig BM data (cross in Fig. 9) and our behavioral estimate in human are in reasonable agreement. However, given the very limited data and their inherent uncertainty, more data are re-

quired to ascertain whether the patterns are indeed similar or whether systematic differences between species exist.

Because of the approximate nature of the measure, it is prudent not to place too much emphasis on the exact values reported here. In particular, for method 1 the mean phase curvature values at 125 and 250 Hz are not significantly different from zero. Nevertheless, the finding that the phase curvature does not scale with CF below about 1000 Hz is very robust. This departure from scaling at low CFs may be related to the relative broadening of the magnitude of the auditory filters at CFs below about 1000 Hz (e.g., Glasberg and Moore, 1990).

## V. SUMMARY

The results provide a first indication of how the phase curvature of the auditory filters varies with level and characteristic frequency (CF) over a wide range. Experiment 1 showed that the phase responses of the auditory filters do not exhibit scaling symmetry at frequencies of 1000 Hz and below. This is consistent with physiological findings in other mammals (Shera, 2001a), as is the lack of an effect of overall level on estimated phase curvature (Shera, 2001b), found in experiment 2. In contrast to findings from auditory-nerve data in the cat (Carney *et al.*, 1999), however, our data show no evidence for a reversal of phase curvature at very low CFs in humans. It is not clear from our data whether scaling symmetry holds above 1000 Hz. Estimates using method 2 suggest a shallow but continuing increase in relative phase curvature at frequencies of 2 kHz and above, whereas the estimates from method 1 seem to be more constant. It has been argued that the frequency glides found in BM impulse responses provide strong constraints for models of cochlear mechanics (de Boer and Nuttall, 1997, 2000a, b; Shera, 2001a). The present data provide similarly strong constraints on models of human auditory filtering. Recent simulations (Oxenham and Dau, 2001) have shown that many of the filters currently used to simulate peripheral auditory processing, such as the gammatone (de Boer and Kruidenier, 1990), the gammachirp (Irino and Patterson, 1997, 2001), or transmission-line models (e.g., Strube, 1985), do not exhibit the phase curvature necessary to predict such data.

## ACKNOWLEDGMENTS

This work was supported by the NIH/NIDCD Grant No. R01 DC 03909 (A.J.O.) and by the Max Kade Foundation and the Deutsche Forschungsgemeinschaft (DFG) (T.D.). The final version of this paper was prepared while the first author was a fellow at the Hanse Institute for Advanced Study in Delmenhorst, Germany. We are grateful to Chris Shera for fruitful discussions and suggestions, and for providing us with the results of his data analysis. Van Summers, Chris Plack, and Jennifer Lentz provided helpful comments on earlier versions of this manuscript. Stephan Ewert provided assistance with the programming.

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